Functions: Transformations of Functions

Most functions that you come across are NOT basic functions.

They have are bigger, smaller, or are not centered at the origin.

We will look at how new functions are built from the basic functions we've already seen.

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Translations: Moving functions around

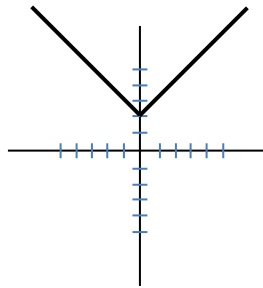Functions are translated by adding or subtracting numbers from the function.

Adding a number on the OUTSIDE of the function: $y = f(x) + d$ moves the graph _____.

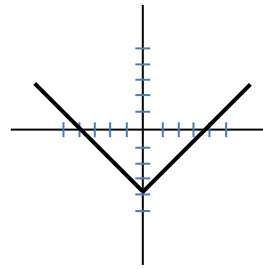Subtracting a number on the OUTSIDE of the function: $y = f(x) - d$ moves the graph _____.

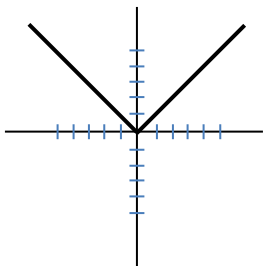For example,m Let $f(x) = y = |x|$ be our basic function.
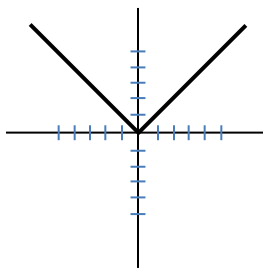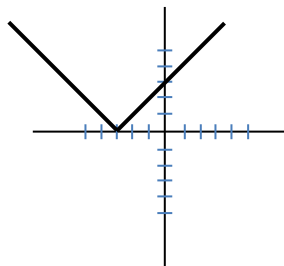

$y = |x|$ $y = |x| + 2$ $y = |x| - 4$

Adding a number on the INSIDE of the function: $y = f(x + c)$ moves the graph right **c** units.
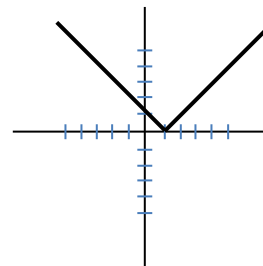
Subtracting a number on the INSIDE of the function: $y = f(x - c)$ moves the graph left **c** units.

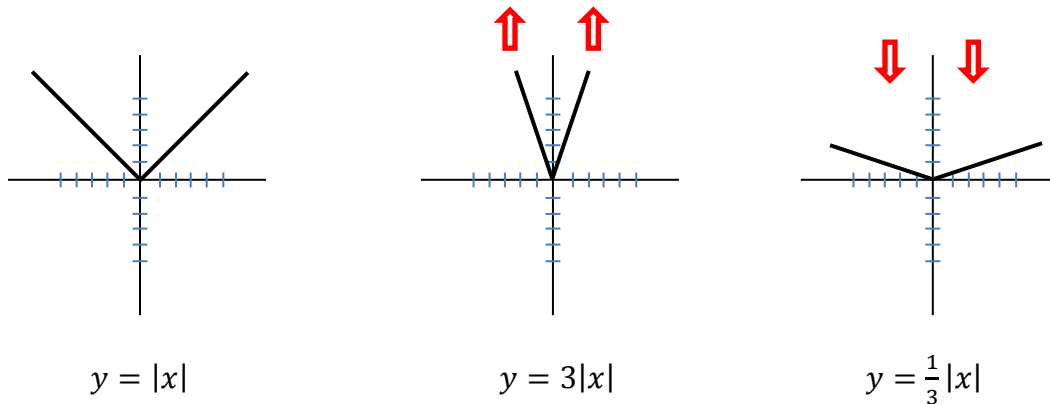$y = |x|$ $y = |x + 3|$ $y = |x - 1|$

Dilations: Changing the size of the graph

Functions are dilated by multiplying or dividing the function by a number.

Multiplying a number on the OUTSIDE of the function: $y = af(x)$ changes the graph's size.

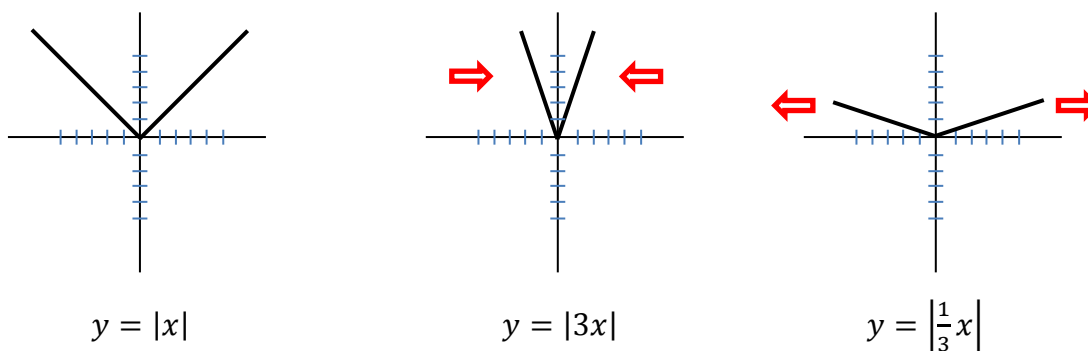IF $a > 1$, the graph is STRETCHED in the VERTICAL direction.

IF $a < 1$, the graph is COMPRESSED in the VERTICAL direction.

$y = |x|$ $y = 3|x|$ $y = \frac{1}{3}|x|$

Multiplying a number on the INSIDE of the function: $y = f(bx)$ changes the graph's size.

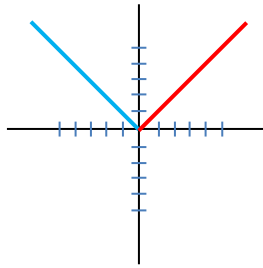IF $b > 1$, the graph is COMPRESSED in the HORIZONTAL direction.

IF $b < 1$, the graph is STRETCHED in the HORIZONTAL direction.
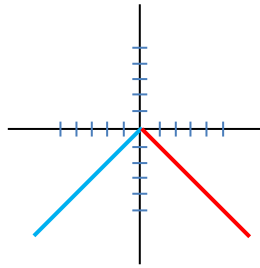

$y = |x|$ $y = |3x|$ $y = \left|\frac{1}{3}x\right|$

Reflections: Flipping the graph over a line.

If you multiply the function by -1, $y = -f(x)$, the graph is REFLECTED over the x-axis.

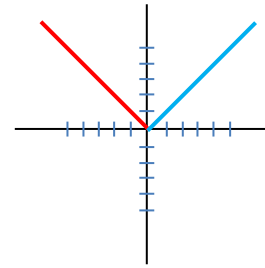If you multiply only the x variable by -1, $y = f(-x)$, the graph is REFLECTED over the y-axis.
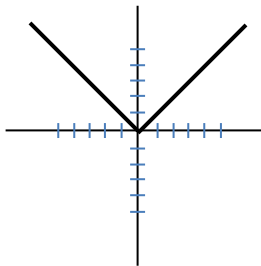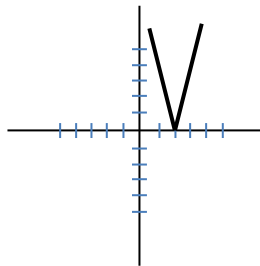
| $y = |x|$ | $y = -|x|$ | $y = |-x|$ |

You can also have combinations of these transformations

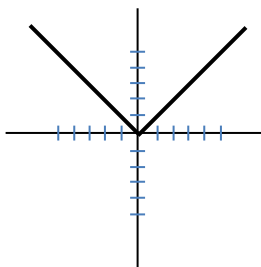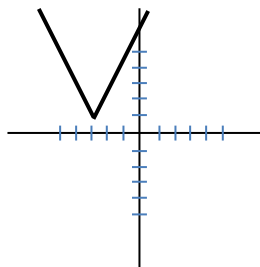Ex: $y = 4|x - 2|$ Stretches vertically by a factor of 4, moves right 2 units.

| $y = |x|$ | $y = 4|x - 2|$ |

Ex: $y = |2x + 6| + 1$ Compresses horizontally by a factor of 2, moves up 1 unit and

$(y = |2x + 6| + 1 = |2(x + 3)| + 1)$ moves left 3 units.

| $y = |x|$ | $y = |2x + 6| + 1$ |